SYNOPSIS

Title : "A STUDY OF SOME PROBLEMS ON ALGEBRAIC STRUCTURE OF RING THEORY"

(a) Introdution

Ring theory is a branch of modern algebra that explores rings- algebraic systems equipped with two operations, addition and multiplication, similar to the way these work for integers. It delves into the internal structure of rings including modules as well as into particular types like group rings and division rings. Special consideration is given to commutative rings and the subject has significant applications in areas such as algebraic geometry and algebraic number theory. The moduli space of complex algebraic curves is a central object in mathematics, with deep applications in mathematical physics, especially through techniques like the Laplace equation. The modular group plays a crucial role here, influencing the geometry of the moduli space, R-matrices, and quantum groups. S.M. Natanson and A.M. Macbeth have examined various facets of modern algebra, including the modular group in homological algebra. We propose to generalize their analyses within a non-commutative framework, focusing on structures such as idempotents and zero divisors.

(b) Work Plan

Connes and Dubois used algebraic methods to deduce properties of both commutative and non-commutative rings. We propose to build a theory of representations, category theory and Galois theory grounded in algebraic geometry and number-theoretic analysis, all consistent with the principles of non-commutative algebra. Our goal is to develop algebraic systems that form the foundation of what is called modern algebra. We have already studied one key piece of this, namely, groups and now aim to introduce and examine ring theory. We propose to extend ring theory, building on the concept of a group, which itself originates from sets of mappings. We propose examine the algebraic properties of the integers and then develop a ring theory, which differs fundamentally from group theory because a ring has two operations,

usually called addition and multiplication. We extend these operations present relevant theorems and prove significant results on ring decompositions. In doing so, we extend the arithmetic facts of the integers to a broader class of algebraic systems specifically rings.

(c) Details of the project with chapterisation

A. Polishchuk established a periodicity theorem in non-commutative algebra that proves useful for addressing injectivity problems in homological algebra. We intend to construct a non-commutative C*-algebra framework tied to Lie groups and Lie algebras. Moreover, we plan to explore the topological aspects of non-commutative theory, particularly via integral dependence, normalization, and non-commutative permutation groups. We have defined the notion of a homomorphism (and its kernel) for rings, drawing on our familiarity with groups. Based on this analogy, we hope to construct in rings something similar to the quotient group formed by dividing by a normal subgroup. And, if we are optimistic, we expect that the homomorphism theorems for groups will carry over in full to the setting of rings. The central goal is to compare the advantages of non-commutative structures with the classical approach using conformal mappings and the modular group. We plan to explore von Neumann algebras and their applications to algebraic surfaces, especially Riemann surfaces. we construct a quotient ring of a ring by an ideal, we will adapt the homomorphism theorems from group theory to the world of rings. Because of the broad applications across mathematics. we propose the research project work is organized into four different chapters are as follows.

Chapter - 1	Properties of Ring and Module theory
Chapter - 2	Structure and classificaiton of finite and algebraic group
Chapter - 3	Properties of Homological Algebra and category theory
Chapter - 4	Non-commutative permuation groups and its impact on cohomolgoy

(d) Hypothesis

The main concepts of algebraic geometry is to examine the geometry of a manifold by using the algebraic properties of a ring of functions on this

manifold. The non-commutative algebraic geometry extends these methods and the geometric intuition to an appropriate class of non-commutative rings. We propose to investigate the subring criterion and illustrate key ring homomorphisms by examining their kernels and images. Another focus is on the group of units in a ring, and the definition of a field. In particular, we will derive the group of units in rings arising from modular representation theory, and then discuss zero-divisors and integral domains. Finally, our main emphasis will be to compare the benefits of non-commutative structures with classical approaches, such as conformal mappings and the modular group and module categories and homological dimensions.

(e) Motivation

The research project to be submitted is purely theoretical in its nature. We have access to relevant books in the Central Library of B. N. M. University. For more recent books and journals, we can consult the National Library's Science Section in Kolkata as well as the Indian Statistical Institute.

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